UNIVERSITÄT
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Faculty of Business Administration
and Economics

IMW

Dependence Uncertainty

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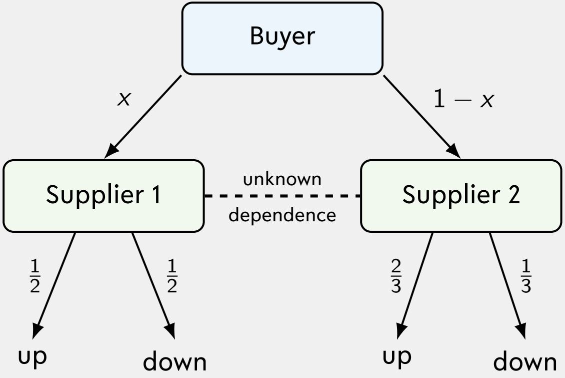
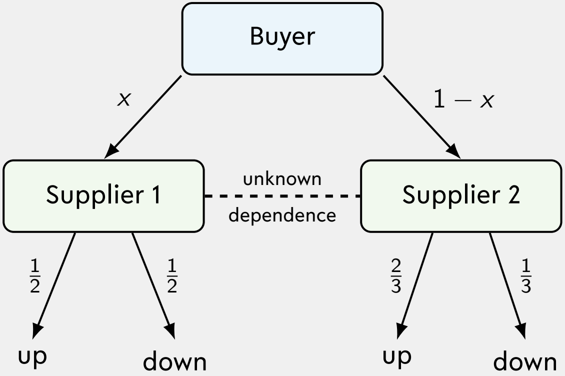
Dual-sourcing procurement with disruption



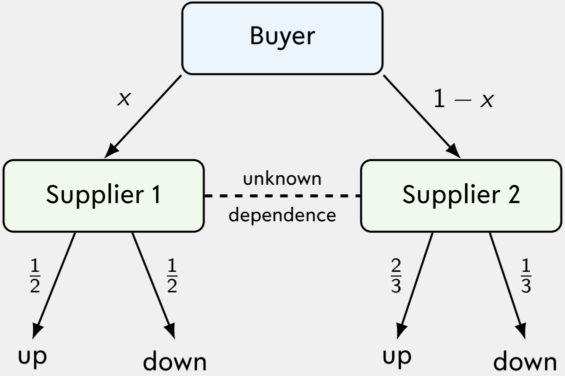
- Sourcing share x to Supplier 1.
- Known individual disruption rates (audits, service histories).
- Dual sourcing enhances reliability and resilience, in case of common shocks (natural disaster, geopolitics).
- Optimal sourcing depends on the unknown joint disruption structure.

Dual-sourcing procurement with disruption



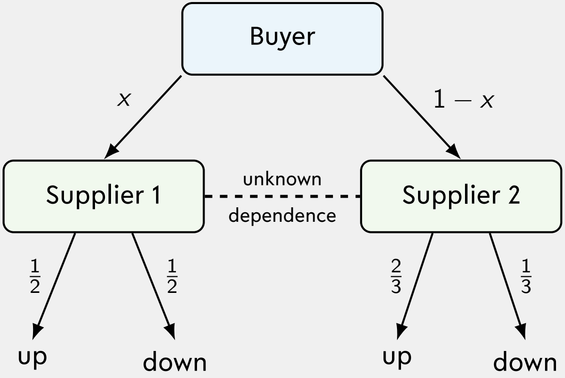
p_{ind}	u_2	d_2
u_1	$\frac{1}{3}$	$\frac{1}{6}$
d_1	$\frac{1}{3}$	$\frac{1}{6}$

Dual-sourcing procurement with disruption



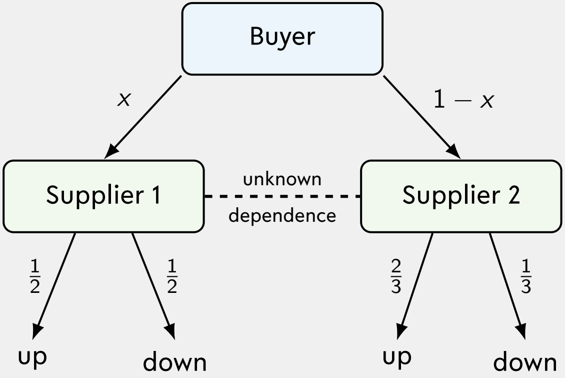
p	u_2	d_2
u_1	$\frac{1}{3} + a$	$\frac{1}{6}$
d_1	$\frac{1}{3}$	$\frac{1}{6}$

Dual-sourcing procurement with disruption



p	u_2	d_2
u_1	$\frac{1}{3}+a$	$\frac{1}{6}-a$
d_1	$\frac{1}{3}-a$	$\frac{1}{6}+a$

Dual-sourcing procurement with disruption



p	u_2	d_2
u_1	$\frac{1}{3} + a$	$\frac{1}{6} - a$
d_1	$\frac{1}{3} - a$	$\frac{1}{6} + a$

- $a \in [-\frac{1}{6}, \frac{1}{6}]$
- p_{ind} for $a = 0$
- $q = \begin{pmatrix} a & -a \\ -a & a \end{pmatrix}$

Dependence Uncertainty Set

Consider a finite state space $\Omega = \Omega_1 \times \ldots \times \Omega_n$ with prescribed $p_i \in \Delta(\Omega_i)$.

Definition

The *dependence uncertainty set* is

$$\mathcal{P} = (\Omega; p_1, \ldots, p_n) := \left\{ p \in \Delta(\Omega) \mid p|_{\Omega_i} = p_i \, \forall i \in \{1, \ldots, n\} \right\}.$$

$\mathcal{P}$ is a compact convex polytope with $p_{\text{ind}} := p_1 \otimes \ldots \otimes p_n \in \mathcal{P}$.

▸ extreme points

▸ dimension

Dependence Structures and Dependence Decomposition

Definition

The set of *dependence structures* is

copulae

$$Q = Q(\Omega) := \{q\colon \Omega \to \mathbb{R} \mid \sum_{\omega_{-i}} q(\omega, \omega_{-i}) = 0 \; \forall \omega_i \in \Omega_i\}.$$

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Proposition

Every $p \in \mathcal{P}$ can uniquely be written as $p = p_{\text{ind}} + q$ with $q \in Q$. Furthermore,

$$\mathcal{P} = \{p_{\text{ind}} + q \mid q \in Q\} \cap \Delta(\Omega).$$

Preferences and Dependence Uncertainty

Consider the set $\mathcal{F}$ of acts $f : \Omega \to X$ (Anscombe and Aumann, 1963).
Today, $X = \mathbb{R}$, $u(x) = x$.

Preferences and Dependence Uncertainty

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Today, $X = \mathbb{R}$, $u(x) = x$.

We propose to say a preference relation $\succeq$ on $\mathcal{F}$ reflects *dependence uncertainty*, if there are marginals $p_i \in \Delta(\Omega_i)$ and an act-dependent ambiguity index $\alpha : \mathcal{F} \to [0, 1]$ such that $\succeq$ is represented by the functional

$$V(f) = \alpha(f) \min_{p \in \mathcal{P}} \int_\Omega f \, \mathrm{d}p + (1 - \alpha(f)) \max_{p \in \mathcal{P}} \int_\Omega f \, \mathrm{d}p,$$

where $\mathcal{P} = \mathcal{P}(\Omega; p_1, \dots, p_n)$.

Dependence-neutral acts

Definition

An act $f \in \mathcal{F}$ is called *dependence neutral*, if for all $p, p' \in \Delta(\Omega)$ with $p|_{\Omega_i} = p'|_{\Omega_i}$ for all $i \in \{1, \dots, n\}$ it holds that $\int_\Omega f \, \mathrm{d}p = \int_\Omega f \, \mathrm{d}p'$. Their set is $\mathcal{F}^{\mathrm{dn}}$.

Equivalently, $\int_\Omega f \, \mathrm{d}q = 0$ for all $q \in Q$.

f_1	u_2	d_2
u_1	x	x
d_1	0	0

$f_1 \in \mathcal{F}_1$

f_2	u_2	d_2
u_1	$1-x$	0
d_1	$1-x$	0

$f_2 \in \mathcal{F}_2$

$f_1 + f_2$	u_2	d_2
u_1	1	x
d_1	$1-x$	0

$f_1 + f_2 \in \mathcal{F}^{\mathrm{dn}}$

Axiomatics

A1. $\succsim$ is complete, transitive and non-trivial.

A2. ($\mathcal{F}^{\text{dn}}$-Archimedean) For all $g \in \mathcal{F}$ and all $f, h \in \mathcal{F}^{\text{dn}}$, if $f \succ g \succ h$, then there exist $\lambda, \kappa \in (0,1)$ such that $\lambda f + (1-\lambda)h \succ g \succ \kappa f + (1-\kappa)h$.

A3. ($\mathcal{F}^{\text{dn}}$-Independence) For all $f, g, h \in \mathcal{F}^{\text{dn}}$, $\lambda \in (0,1)$

$$f \succsim g \iff \lambda f + (1-\lambda)h \succsim \lambda g + (1-\lambda)h.$$

A4. ($\mathcal{F}^{\text{dn}}$-Dominance) For all $f \in \mathcal{F}$, $g \in \mathcal{F}^{\text{dn}}$

$$f(\omega) \succeq (\preceq) g(\omega)\ \forall \omega \implies f \succsim (\precsim) g.$$

Axiomatic Representation of Dependence Uncertainty

Theorem

A preference relation $\succcurlyeq$ on $\mathcal{F}$ fulfills satisfies Axioms 1—4 if and only if there exist an ambiguity index $\alpha\colon \mathcal{F} \to [0,1]$ and $p_1, \dots, p_n \in \Delta(\Omega_1), \dots, \Delta(\Omega_n)$ such that $\succcurlyeq$ is represented by the functional

$$V(f) = \alpha(f) \min_{p \in \mathcal{P}} \int_\Omega f \, \mathrm{d}p + (1 - \alpha(f)) \max_{p \in \mathcal{P}} \int_\Omega f \, \mathrm{d}p,$$

where $\mathcal{P} := \mathcal{P}(\Omega; p_1, \dots, p_n)$.

Compatibility to other uncertainty representation

- Maxmin expected utility (MEU) (Gilboa and Schmeidler, 1989, JME)

$$V_{\text{MEU}}(f) = \min_{p \in \mathcal{C}} \int_{\Omega} f \, \mathrm{d}p \qquad \text{where } \mathcal{C} \subseteq \mathcal{P}.$$

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- Variational preferences (V) (Maccheroni, Marinacci, and Rustichini, 2006, ECMA; Hansen and Sargent, 2001, AER)

$$V_{\text{VP}}(f) = \min_{p \in \Delta\Omega} \left\{ \int_{\Omega} f \, \mathrm{d}p - c(p) \right\} \qquad \text{where } c(p) = \infty \text{ on } \Delta(\Omega) \setminus \mathcal{P}.$$

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- Smooth ambiguity (Klibanoff, Marinacci, and Mukerji, 2005, ECMA)

$$V_{\text{smooth}}(f) = \int_{\Delta(\Omega)} \phi \left(\int_{\Omega} f \, \mathrm{d}p \right) \mathrm{d}\mu \qquad \text{where } \operatorname{supp}(\mu) \subseteq \mathcal{P}.$$

Dependence sensitive act

f	u_2	d_2
u_1	1	
d_1		

Dependence sensitive act

f	u_2	d_2
u_1	1	$x + \frac{1}{2}(1-x)$
d_1	$\frac{1}{2}x + (1-x)$	

Dependence sensitive act

f	u_2	d_2
u_1	1	$x + \frac{1}{2}(1-x)$
d_1	$\frac{1}{2}x + (1-x)$	$\frac{3}{20} + \frac{4}{5}x - \frac{1}{5}x^2$

Dependence sensitive act

f	u_2	d_2
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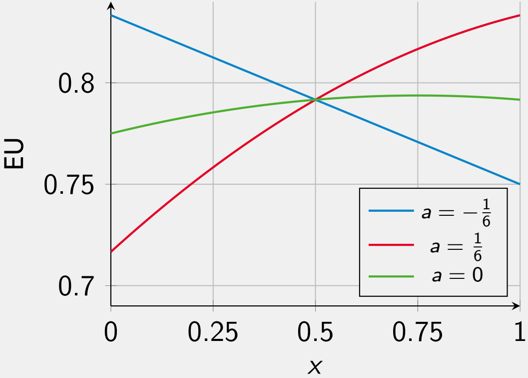
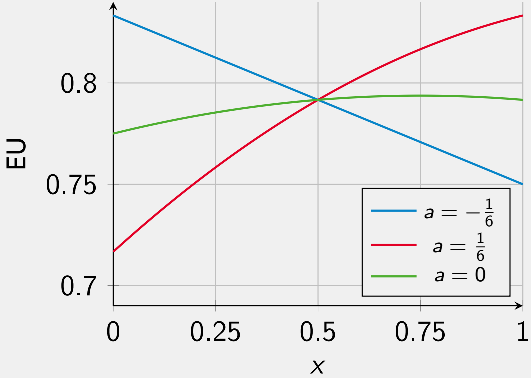


▸ state-wise utility

Dependence sensitive act

f	u_2	d_2
u_1	1	$x + \frac{1}{2}(1-x)$
d_1	$\frac{1}{2}x + (1-x)$	$\frac{3}{20} + \frac{4}{5}x - \frac{1}{5}x^2$

- $a = -\frac{1}{6}$ $\rightarrow x^* = 0$
- $a \in \{0, \frac{1}{6}\}$ $\rightarrow x^* = 1$
- $a \in [-\frac{1}{6}, \frac{1}{6}]$ & MEU $\rightarrow x^* = \frac{1}{2}$



▸ state-wise utility

Dependence Neglect — I

Humans neglect dependence

- when making financial decisions (Eyster and Weizsäcker, 2010; Eyster and Weizsäcker, 2016; Kallir and Sonsino, 2009),
- when forming beliefs (Enke and Zimmermann, 2019, REStud).

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Formal representation

$$V(f) = \int_{\Omega} f \, \mathrm{d}p_{\mathrm{ind}}.$$

➤ We make dependence neglect testable by an axiom!

Dependence Neglect — II

A5. (Subspace Separation) For all $f_i, g_i \in \mathcal{F}_i$, $y \in X$, non-null $E_{-i} \subseteq \Omega_{-i}$,

$$f_i \succcurlyeq g_i \iff f_{i[E_{-i}]}y \succcurlyeq g_{i[E_{-i}]}y.$$

f_1	u_2	d_2
u_1	1	1
d_1	0	0

$\succcurlyeq$

g_1	u_2	d_2
u_1	0.8	0.8
d_1	0.4	0.4

$\iff$

$f_{1[u_2]}y$	u_2	d_2
u_1	1	y
d_1	0	y

$\succcurlyeq$

$g_{1[u_2]}y$	u_2	d_2
u_1	0.8	y
d_1	0.4	y

Dependence Neglect — II

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$$f_i \succcurlyeq g_i \iff f_{i[E_{-i}]}y \succcurlyeq g_{i[E_{-i}]}y.$$

Theorem

If $\succcurlyeq$ is MEU with prior set $\mathcal{C} \subseteq \mathcal{P}(\Omega; p_1, \dots, p_n)$, Axiom 5 holds if and only if $\succeq$ is SEU with respect to the independent product $p_{\text{ind}} = p_1 \otimes \dots \otimes p_n$.

Definition

We say that $\succcurlyeq$ is *more dependence averse* than $\succcurlyeq'$ if for all $f \in \mathcal{F}$ and $g \in \mathcal{F}^{\mathrm{dn}}$

$$f \succcurlyeq g \implies f \succcurlyeq' g.$$

Comparative Dependence Uncertainty and Dependence Premia

Definition

We say that $\succcurlyeq$ is *more dependence averse* than $\succcurlyeq'$ if for all $f \in \mathcal{F}$ and $g \in \mathcal{F}^{\mathrm{dn}}$

$$f \succcurlyeq g \implies f \succcurlyeq' g.$$

Definition

The *dependence premium* of $f \in \mathcal{F}$ is the value $\mathrm{DP}(f)$ satisfying

$$\int_{\Omega} f - \mathrm{DP}(f)\,\mathrm{d}p_{\mathrm{ind}} = V(f)$$

Comparative Dependence Uncertainty

Proposition

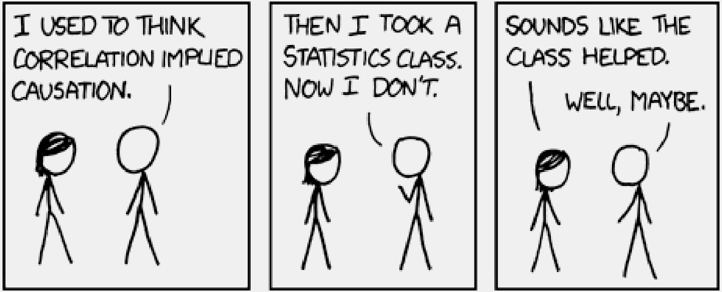
For dependence uncertainty preferences, $\succcurlyeq$ is more dependence averse than $\succcurlyeq'$ iff one of the following statements hold:

(i) $p_i = p_i'$ *for all* $i \in \{1, \dots, n\}$ *and* $\alpha(f) \geq \alpha'(f)$ *for all* $f \in \mathcal{F} \backslash \mathcal{F}^{\mathrm{dn}}$.

(ii) $\mathrm{DP}(f) \geq \mathrm{DP}'(f)$ *for all* $f \in \mathcal{F}$.

(iii) $\mathcal{C}' \subseteq \mathcal{C}$ *(MEU)* (iv) $c^* \leq c^{*\prime}$ *(VP)* (v) $\phi = h \circ \phi'$, *h concave (smooth)*

plots



Bauch and Hartmann, 2026

Thank You!

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Extreme points of $\mathcal{P}$

back

Theorem

The following are equivalent for a probability $p \in \mathcal{P}$ and any divergence $\mathrm{D}(\cdot \parallel \cdot)$ with infinitely decreasing slope in 0.

(i) *p is an extreme point of $\mathcal{P}$.*

(ii) *p is maximally zero.*

(iii) *p is a strict local maximizer of the divergence $\mathrm{D}(\cdot \parallel p_{\mathrm{ind}})$.*

E.g., φ-divergence: $\varphi'(0) = -\infty$. Examples: α-divergence for $\alpha \in [0, 1]$, including (generalized, reversed) Kullback-Leibler divergence, Jeffrey's divergence, Jensen-Shannon divergence, squared Hellinger distance, Neyman χ^2-divergence (aka reverse Pearson).

Dimension of $\mathcal{P}$

back

Rectangular operations are dependence structures q with support on a rectangle:

$$
\begin{tikzcd}
(\omega_i^*, \omega_{-i}^*) & (\omega_i^*, \omega_{-i}^\dagger) \arrow[l, "a"'] \\
(\omega_i^\dagger, \omega_{-i}^*) \arrow[r, "a"] & (\omega_i^\dagger, \omega_{-i}^\dagger)
\end{tikzcd}
$$

> **Proposition**
>
> *Rectangular operations form a spanning system of Q. Setting* $m_i := \#\,\mathrm{supp}(p_i)$, *the dimension of* $\mathcal{P}$ *is equal to* $\prod_{i=1}^{n} m_i - \sum_{i=1}^{n}(m_i - 1) - 1$.

Dependence Structures and Copulae — I

- Correlation is often measures by a *copula* in statistics, cf. Nelsen, 2006.
- A copula is a joint distribution of $[0, 1]^n$ with *uniform* marginals.
- Copula theory works only for real-valued random variables
  - ➤ Dependence structures work on arbitrary state spaces.
  - ➤ Copulae can indeed be identified as special dependence structures.

▸ dependence structures

Dependence Structures and Copulae — II

C	$\frac{2}{3}$	1
$\frac{1}{4}$	$\frac{1}{6}+x$	$\frac{1}{4}$
1	$\frac{2}{3}$	1

$C - C_{\text{ind}}$	$\frac{2}{3}$	1
$\frac{1}{4}$	x	0
1	0	0

$p_{\text{ind}} + q$	$\frac{2}{3}$	1
$\frac{1}{4}$	$\frac{1}{6}+x$	$\frac{1}{12}-x$
1	$\frac{1}{2}-x$	$\frac{1}{4}+x$

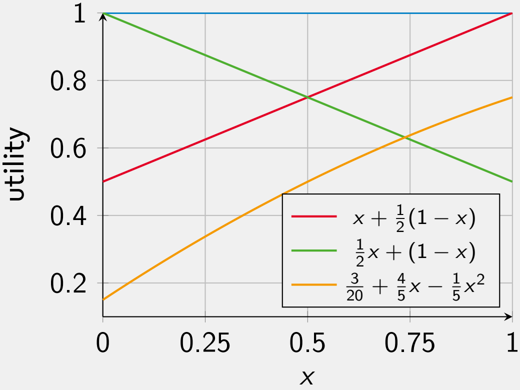
q	$\frac{2}{3}$	1
$\frac{1}{4}$	x	$-x$
1	$-x$	x

A copula C and its corresponding dependence structure q for $\Omega_1 = \{\frac{1}{4}, 1\}$, $\Omega_2 = \{\frac{2}{3}, 1\}$ with $x \in [-\frac{1}{6}, \frac{1}{12}]$.

dependence structures

Dependence sensitive act — State-wise Utility

f	u_2	d_2
u_1	1	$x + \frac{1}{2}(1 - x)$
d_1	$\frac{1}{2}x + (1 - x)$	$\frac{3}{20} + \frac{4}{5}x - \frac{1}{5}x^2$



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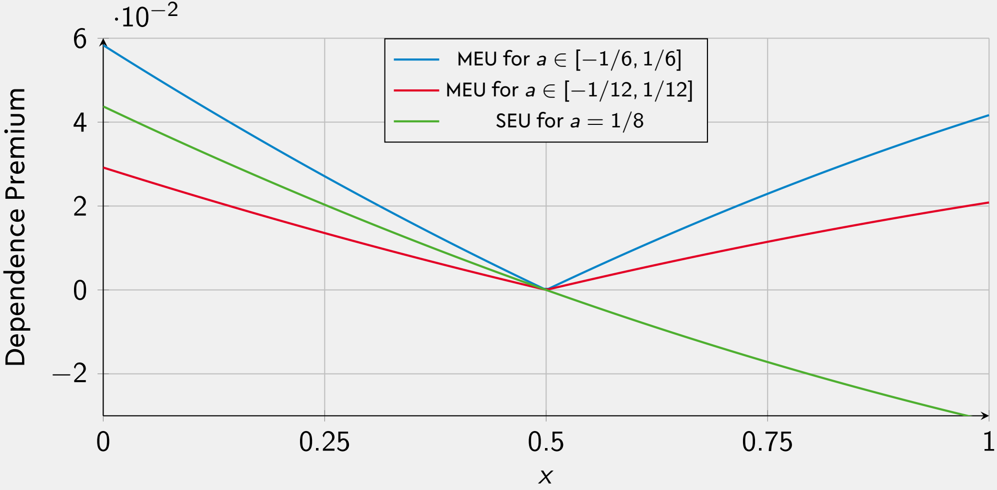
Dependence Premia for Different Preferences



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